Dynamic Hedging by a Large Player

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Large Player & liquidity

- Large traders behave de facto as liquidity providers
  - They are net writers of options (sell 50 % than they buy Kambhu (1998))
  - Value investors (or fundamentalists) & Trend followers (or chartists) enter into contracts with large traders because they hold sufficient liquid assets to meet their liquidity needs

- But they hardly transact
  - without putting pressures on prices
    - orders are filled at a price that is shifted from the previous one

      ⇒ a price formation rule relating the net order to the new price, through a market price impact function
  - directly impacting liquidity (degree to which *large* size transactions can be carried out in a *timely* fashion with minimal *impact* on prices)
  - hence empirically observed feedback effects on mid term interest rates & equity derivatives
  - ⇒ « Paradox » of the Portfolio Insurance Strategy

⇒ *Endogeneous Bid Ask Spread due to inventory holding costs*

⇒ specific *hedging* strategies
  - limit inventory holding costs through partial hedging & managing the Greeks

⇒ *strategic hedging*
  - trading volatility spreads
  - using insider information

Market framework

- Interaction of one "large trader" whose action affects prices and many price taking "small traders”
- The usual no arbitrage condition (Delbaen and Schachermayer (1994)) doesn’t apply

- ⇒ continuous time version of Jarrow (JFQA 1994): “No Market Manipulation Strategies“
- ⇒ Our approach also differs by the fact that the large trader is better informed.
- *The additional required assumptions are:*
  - The underlying asset price process is independent of the large trader's past holdings.
  - Real wealth
    - the value as if the holdings were liquidated
  - Synchroneous Markets Condition
    - prices adjust instantaneously across underlying and derivative markets.
    - otherwise the large trader may use information mismatches between them to post riskless profits.
  - Absence of corners
    - the combined effective holding of the underlying asset and the derivative must not exceed the net supply of the underlying asset.

Outline of the Talk

- I From feedback hedging effects to the bid-ask spread
- II Partial hedging strategies
- III Managing the Greeks
- IV Strategic hedging

From the hedging feedback effects to the bid offer spread

Incorporating the hedging feedback effects

- Whereas small traders are roughly balanced between supply and demand sides, large dealers are empirically found to be net writers of options
- ⇒ additional process = number of underlying assets held by the large trader

$$d\tilde{S}_t = \sigma_t \tilde{S}_t dW_t + \rho_t \lambda\left(\tilde{S}_t\right)\tilde{S}_t d\alpha_t$$

⇒ *non linear feedback effects (Frey, Stochastic Finance, 1998)* ⇒ non linear PDE

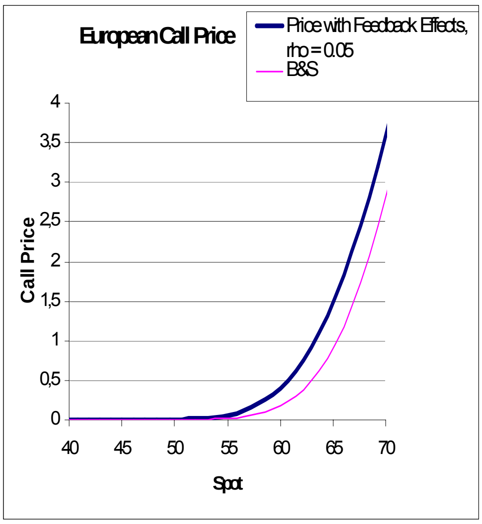
- net writers of options ⇒ neutralize risk by replicating synthetically option positions

$$\begin{cases} u_t\left(t,\tilde{S},\gamma\right) + \frac{1}{2}\frac{1}{\left(1+\rho\lambda\left(\tilde{S}\right)\tilde{S}u_{\tilde{S}\tilde{S}}\left(t,\tilde{S},\gamma\right)\right)^2}\sigma^2\tilde{S}u_{\tilde{S}\tilde{S}}\left(t,\tilde{S},\gamma\right) = 0 \\ u\left(T,\tilde{S},\gamma\right) = nh\left(\tilde{S}\right) \end{cases}$$

⇒ Paradox of the Portfolio Insurance Strategy

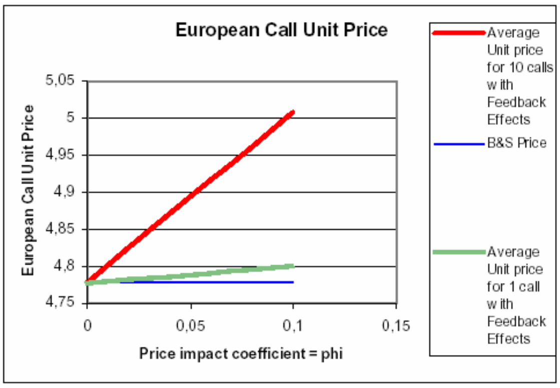
Ex: the large trader sells a European call ⇒ buys underlying assets to hedge ⇒ underlying price rises ⇒ short delta is more negative ⇒ gamma < 0 ⇒ feedback volatility rises ⇒ European call price rises (not decreases !)

⇒ *Bid & offer prices with corresponding feedback volatilities*

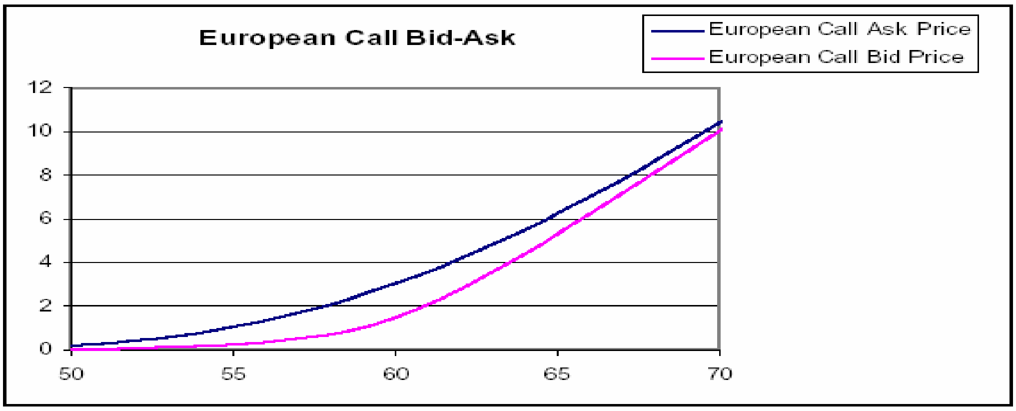


Bid-offer & inventory costs

- The replicating cost grows more than linearly with respect to the number of options held (Carr, Geman, Madan, JFE, 2001)



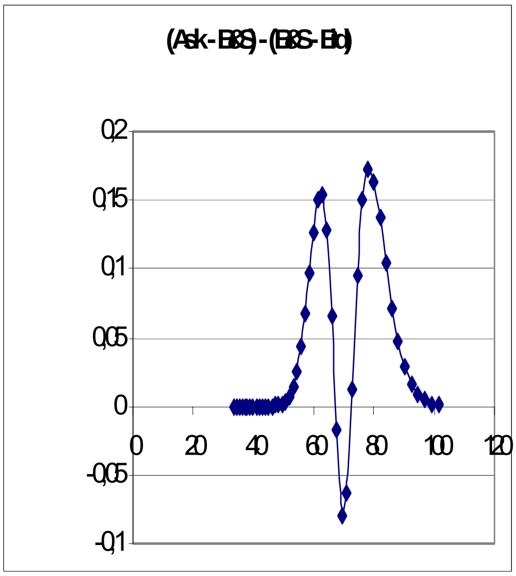
Dynamic hedging ⇒ Bid-offer spread



Consistent with empirical statistical features

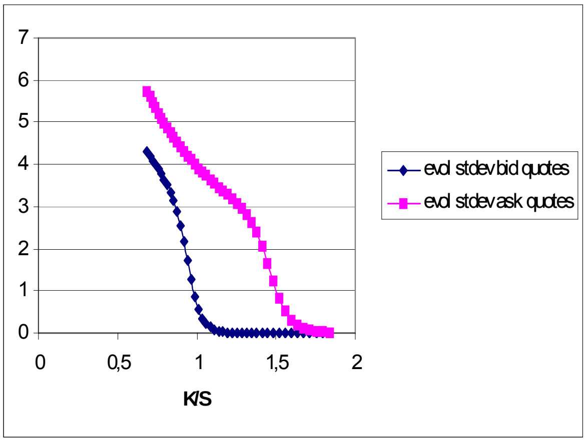
- Asymmetry (option price closer to the bid) increases in and out the money

- The ask changes more than the bid, idem for their standard deviation, and the standard deviation of their changes



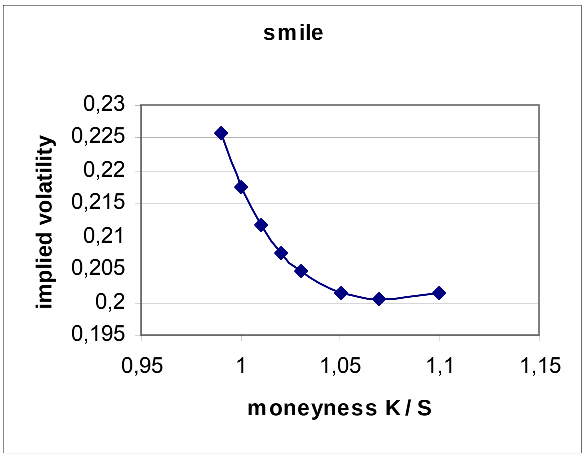
Endogenous Volatility Smirk

- Both Quotes Standard devations decrease w.r.t. the moneyness (K/S) for a call (and the reverse for a put), implying a volatility smile



- It is created through an endogenous variable volatility in a complete market.

  ⇒ Smile or skew through the specification of λ



$$\lambda(S) = 1 + (S - S_0)^2 (a_1 1_{\{S \le S_0\}} + a_2 1_{\{S \ge S_0\}})$$

- Extensions for vega hedging strategies

  1) Stochastic volatility (perturbation methods)

  2) Business time liquidity pricing with n(t) = open interest

$$dS_t = \sigma_t S_t dW_{n(t)} + \rho_t \lambda(S_t) S_t d\alpha_t$$

$$dn(t) = a(t)dt$$

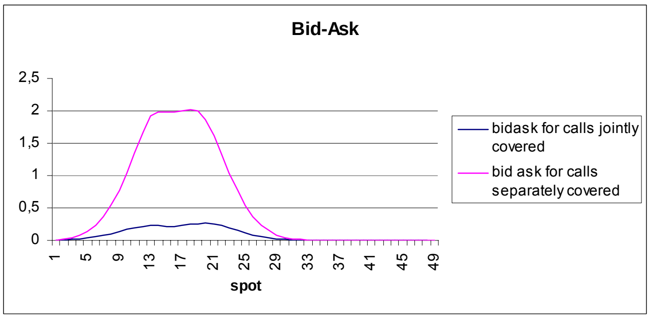
$$da_t = \beta(t,a)dt + \nu(t,a)dZ(t)$$

Partial Hedging Strategies

There is a trade-off: the goal is to minimize the bid-ask spread, in order to attract more customers, while keeping a reasonable hedging ratio

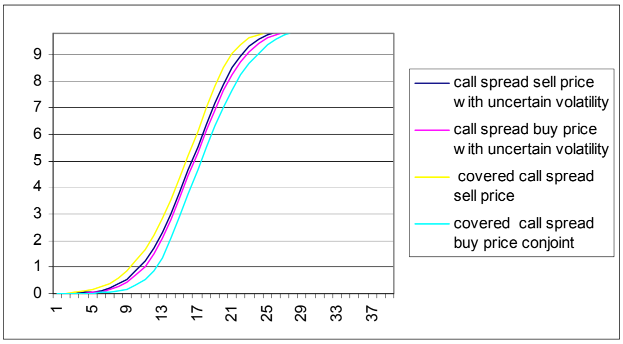
Partial hedging

- The large player decides to limit thedynamic replication to her residual exposure
- But her portfolio is not quoted ⇒ volatility is uncertain ⇒ she uses the two previous "feedback" volatilities.



$$\begin{cases} \frac{\partial}{\partial t} p_t^\gamma(s) + \frac{1}{2}\frac{\partial^2}{\partial s^2} p_t^\gamma(s)\gamma^2(t,s)s^2 = r p_t^\gamma(s) \\ \gamma(t,s) = \begin{cases} \sigma^+ \; si \; \frac{\partial^2}{\partial s^2} p_t^\gamma(s) \geq 0 \\ \sigma^- \; si \; \frac{\partial^2}{\partial s^2} p_t^\gamma(s) < 0 \end{cases} \\ (s-K_1)^+ - (s-K_2)^+ = p_T^\gamma(s) \end{cases}$$

Example: Call spread : long one call (strike K1) and short another (strike K2)



But hedging error may be high

$$e_t = \frac{1}{2}\int_0^T \frac{\partial^2}{\partial s^2} p_t^\gamma(s) s_t^2(\gamma_t^2 - \sigma_t^2)dt + \int_0^T \sigma_t s_t \left[\Delta(t) - \frac{\partial}{\partial s} p_t^\gamma(s)\right] dW_t$$

State dependent strategies (1)

- Excessive transaction costs linked with trading frequency ⇒ hedge only when the expected price movement is large enough to .

- Define

$$X_t = (S_t - s)\exp(-rt)$$

Example: she buys options (puts) only at a date defined by a sequence of 3 conditions

- when the underlying S asset falls under a predefined “alarm” level s. Define

$$g(t) \coloneqq \sup\{s \leq t : X_s = 0\}$$

- if S remains there a certain period of time, i.e. for a given constant α

$$\tau_\alpha \coloneqq \inf\left\{t > 0 : t - g(t) \geqslant \frac{\alpha^2}{2} \; where \; X_s \leq 0 \; \forall s \in (g(t^-), t)\right\}$$

and if it falls further down under a threshold level

$$\tau \coloneqq \inf\left\{t > 0 : X_t = 2X_{\tau_\alpha} \; where \; X_s < 0 \; \forall s \in (\tau_\alpha, t)\right\}$$

State dependent strategies (2)

- Define $$Y_t = \begin{cases} X_t \ \ for\ t < \tau_\alpha \\ 2X_{\tau_\alpha} - X_t\ for\ t \geq \tau_\alpha \end{cases}$$

$$\tilde{\mathcal{G}}_t \coloneqq \sigma\{sign(Y_s); s \leq t\}$$

- when the large player always hedges, the hedging cost is the previous price with feedback effects V_0^ξ

- when the threshold value scenario is never activated from time 0 to time T, i.e. on [τ>T], the hedging cost is

$$NHC_0 = 1_{[\tau>0]}\mathbb{E}\left[1_{[\tau>T]} \exp\left(-\int_0^T r_u du \right) h(S_T)|\mathcal{G}_0 \right]$$

Assuming interest rates are deterministic, and that the latter stopping time has intensity λ, we use Azéma-Yor (1978), in the case of a put option

$$NHC_0 = \left(1 - \left(\mathbb{E}[1_{[\tau_\alpha \leq T]}] - \mathbb{E}\left[\frac{\alpha/\sqrt{2}}{T-\bar{g}_{\tau\alpha}} 1_{[\tau_\alpha < T]} \right]\right)\right) P_0$$

$$P_0 = S_0 N(d_1) - K\exp\left(-\int_t^T r_u du \right) N(d_2)$$

- So on [τ<T], the hedging cost is

$$HC_0 = V_0^\xi - NHC_0$$

Managing the Greeks

with respect to

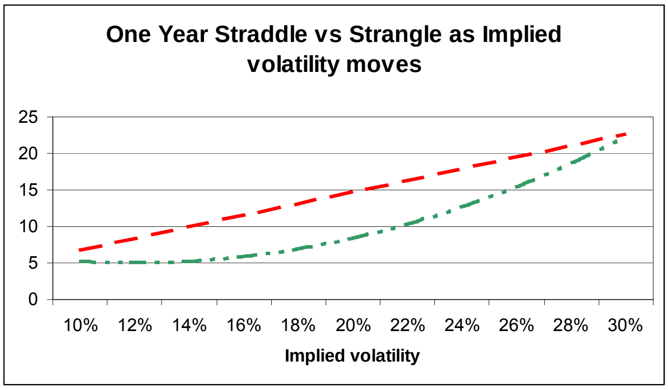
-the net position to be hedged

-the feedback volatility

Managing the gamma

In the short-term the "feedback" volatility is higher or lower than the usual one

- ⇒ buy vega convexity
  - short 100 straddle & long 70-130 strangles,
  - buy low vol ATM and sell high vol OTM options, resulting in short volatility; e.g. butterfly
  - buy variance swaps/sell vol swaps
  - New retail products: Napoleon, Reverse Cliquets, Worst Of Binary…



- Structural long-term short gamma and the corresponding rise in volatility increase the probability of a negative P&L. In the case of a call $C(T)-C(0)=\int_0^T \frac{1}{2}\left(v^2_{t(\text{realized})}-\sigma^2_{t(\text{sold})}\right)S_t^2\frac{\partial^2 C}{\partial S^2}(t,S,\gamma)dt$

  ⇒ should compensate through buying long-term volatility, such as
  - long-term out-of-the-money options
  - variance swaps

  These instruments are priced with a lower volatility by the other small players ⇒ makes profits for the large trader.

Strategic dynamic hedging

Arbitrage opportunities

- For small traders the dynamics is $\frac{d\tilde{S}_t}{\tilde{S}_t} = \sigma_t dW_t$ so the filtration is

$$\mathcal{F}_t = \sigma(W_s, s \le t)$$

- The large trader is in addition aware of the fact that the process returns to its Gaussian dynamics when she stops hedging at time T

$$\frac{dY_t}{Y_t} = d\tilde{Z}_t = \sigma'_t d\bar{W}_t$$

with $(\bar{W})_{0 \le t \le 1}$ independent of W under a suitable « martingale preserving measure » (Amendiger (1999)) based on the assumption

$$P[G \in B | \mathcal{F}_t] \sim P[G \in B]$$

- Which requires the introduction of an enlarged filtration

$$H_t = \mathcal{F}_t \vee G_t \qquad G_t = \sigma\left(\tilde{Z}_t\right)$$

$\Rightarrow$ arbitrage opportunities through the difference between

$$E_{Q_T^G}[(S_T - K)^+]$$

and

$$E_{Q_T^G}\left[\left(\tilde{Y}_T - K\right)^+\right]$$

Hidden hedging strategies

Buying or selling an appropriate quantity of options in such a way that the asset price remains a martingale, so that she remains undiscovered

We revisit Back (1992) seminar paper and exhibit the following optimal strategy α_t^* for the large trader, and thus *the optimal number of options* for the large trader to hold if she wants to delta hedge at the same time (in which case $\alpha_t = \frac{\partial u(\gamma, t, \tilde{S})}{\partial \tilde{S}_t}$)

- Full information

$$\rho_t \lambda_t \left(\tilde{S}_t\right) d\alpha_t^* = \rho_t \lambda_t \left(\tilde{S}_t\right) \frac{\partial^2 u}{\partial S^2}\left(\gamma_t^*, \tilde{S}_t, t\right) = \frac{\tilde{Z}_T - \tilde{X}_t}{T-t} \sigma_t^2 dt$$

- Increasing information

$$\rho_t \lambda_t \left(\tilde{S}_t\right) \frac{\partial^2 u}{\partial \tilde{S}^2}\left(\gamma_t^*, \tilde{S}_t, t\right) = \frac{\tilde{Z}_t - \tilde{X}_t}{\int_0^1 \left(T - \sigma^2(u)\right) du} \sigma_t^2 dt$$

⇒ smoother strategy, minimizing hedging costs

Conclusion

Main contributions

- Extension of the non linear market impact literature
  - Consistent with the volatility smile
- Introduction of an endogenous bid-offer spread
  - Induced by inventory holding cost
  - Consistent with emprically observed statistical properties
- Devising specific hedging strategies
  - Under different scenarios
  - Involving several derivatives and risks
- Pricing asymmetric information
  - Thanks to the specific knowledge of the large trader
  - Inducing price spread and hidden strategies

Extensions

- Further pricing applications with Path dependent options

- Dealing further with Vomma & Vanna hedging

- Manipulation strategies
  - Going beyond the « No Market Manipulation Strategy » assumption of Jarrow